Controlling Chaos by Periodic Perturbations in Nonisothermal Fluidized-Bed Reactor

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A two-phase model of a nonisothermal fluidized-bed catalytic reactor with consecutive exothermic reactions makes it possible to control the chaotic behavior by small parametric perturbations of an input variable with suitable frequency and amplitude. By periodic forcing of the feed temperature the chaotic behavior of the autonomous system can be controlled and become periodic even for small amplitudes of forcing. For constant frequency with change in forcing amplitudes the behavior of the system alternates between periodic and chaotic regimes via period doubling and period adding bifurcation mechanisms. When the amplitude is fixed, regular patterns also appear with small changes in the forcing frequency. Appropriate selection of the forcing frequency can improve the yield of the desired intermediate component in the consecutive reactions network $A \rightarrow B \rightarrow C$.

Introduction

Nonlinear dynamical systems can exhibit a variety of behavior depending on the values of the system operating parameters. For certain values of the parameters, a nonlinear system can exhibit simple or complex (quasi-periodic, as well as strange chaotic and nonchaotic) oscillations. Chaotic behavior shows extreme sensitivity to initial conditions, i.e., nearby process trajectories diverge exponentially. This characteristic known as the "butterfly effect" (Lorenz, 1963) is a formidable obstacle to an adequate control of chaotic systems. In recent years a number of researchers have explored ideas for the control of chaotic behavior. Shinbrot et al. (1993) provided a review of some of these techniques. One aspect of this research is the application of resonant periodic parametric perturbations to stabilize unstable periodic orbits (Alekseev and Loskutov, 1987; Lima and Pettini, 1990; Li et al. 1992; Shinbrot, 1994). It should be noted that the effects associated with these perturbations are generally difficult to predict; nevertheless, this technique is easy to implement and in some cases dramatic changes have been recorded in the behavior of the chaotic system. Lima and Pettini (1990), for instance, used the dynamics of a Duffing-Holmes oscillator to show that small parametric perturbation of suitable frequency can bring the system from chaos to a regular regime. Application of this technique to stabilize a chaotic attractor or, at least, to make it periodic seems to be a promising approach.

In this article we consider the case of the nonisothermal fluidized-bed catalytic reactor with consecutive exothermic reactions $A \rightarrow B \rightarrow C$ under a conventional proportional-integral (PI) controller. It is shown that starting from a chaotic regime for the autonomous, i.e., unforced system it is possible to obtain regular patterns by periodic perturbations of the feed temperature.

The effect of feed disturbances on the behavior of systems having periodic and quasi-periodic autonomous attractors has been investigated by many authors (Mankin and Hudson, 1984; Hoffman and Schadlich, 1986; Silveston et al., 1986; Cordonier et al., 1990; Tambe and Kulkarni, 1993). It was clearly demonstrated that for certain values of amplitude and frequency of the input variable the periodic and quasi-periodic attractors of the autonomous system can turn into chaotic attractors. The importance of these results lies in the fact that some of these changes in the fundamental characteristics of the attractor occur at very small values of the forcing amplitude which are almost impossible to avoid in practice.

In this article the opposite problem is tackled, i.e., where starting from a chaotic attractor of the autonomous system it

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is shown that nonchaotic behavior can be obtained by appropriate forcing of the feed temperature. The effects of the forcing on the yield of the desired product B in the reaction network $A \to B \to C$ is also investigated.

It should be noted that chaotic behavior in the fluidized bed has been reported experimentally in the literature (Daw and Halow, 1993; Daw et al., 1995a). Daw et al. (1995b) have shown in their experimental study of the hydrodynamics of a fluidized bed that the large-scale collective movement of particles, i.e., slugging has many of the signature characteristics of chaos. The authors also investigated how the sensitivity of the slugging bed to small perturbations, along with a feedback control, can be used to enhance and disrupt slugging.

In this article we investigate the control of the chaotic behavior that emanates not from the bed hydrodynamics but rather the reactions taking place in the reactor.

Previous Work

The dynamic behavior of the automomous model of the nonisothermal fluidized-bed reactor with the catalytic exothermic reaction $A \to B \to C$ was investigated by Elnashaie and co-workers (1977, 1995). The unit is shown in Figure 1. The states used to describe the reactor model are the concentrations of the reactant A and product B, and the temperature of the reactor dense phase.

The autonomous, i.e., unforced system is known to exhibit steady-state multiplicity. The model investigated in this article has three steady states, a low-temperature (quenched) steady state, a high-temperature (burn-out) steady state, and a third steady state which is the desired operating point as it occurs at a physically realizable temperature corresponding to the maximum yield. Since this desired steady state is unstable (saddle type), a feedback control system is needed for the operation of the reactor.

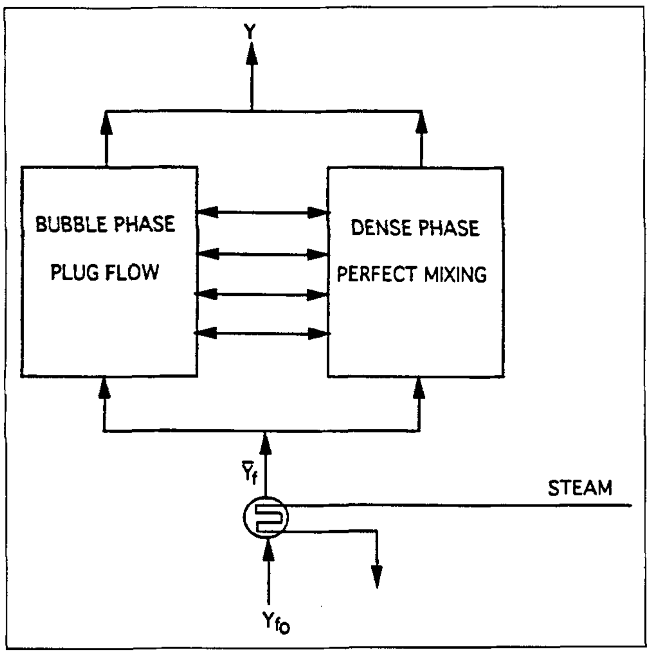

Figure 1. Two-phase fluidized-bed reactor.

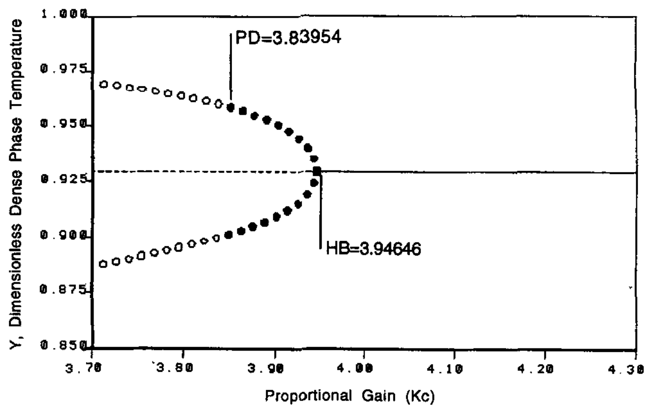

Figure 2. Bifurcation diagram Y vs. K_c when the reactor is under proportional-integral control.

The integral mode gain is $K_i = 0.05$. The integral action destroys the multiplicity of the uncontrolled reactor. The periodic branch emanating from the Hopf bifurcation point (HB) point loses its stability through period doubling (PD) sequence. Solid stable branch; dashed unstable branch.

It is worth mentioning that this situation has practical industrial applications. In many petroleum refining industry reactions, industrial fluidized-catalytic cracking (FCC) units are generally operated at the unstable steady state that gives maximum gasoline yield (Edwards and Kim, 1988). Other examples of the occurrence of this situation in petrochemical reactions are the partial oxidation of o-xylene to phthalic anhydride (Elnashaie et al., 1990) and the oxidative dehydrogenation of ethylbenzene to styrene (Elnashaie et al., 1991).

Elnashaie and Abashar (1994) investigated the chaotic behavior of the periodically forced reactor when it is under a simple proportional controller. The investigation, however, was carried out with a two-component reactor model. The original three component system was reduced to a two-component system by considering the dynamics of the product B to be very fast. The authors showed that period doubling and chaos occur for very small amplitudes. Various mechanisms of transition between periodic windows and chaotic regimes were observed and analyzed.

In this article the original three-component system is investigated when it is under a conventional proportional-integral (PI) controller and when the feed temperature is periodically forced. The autonomous controlled system was investigated by Ajbar and Elnashaie (1994). The continuity diagram of the PI controlled reactor is shown in Figure 2. It can be seen that the integral mode of the controller destroys the steady-state multiplicity. The continuity diagram of the system shows one static branch that corresponds to the desired steady state. The destabilizing effect of the integral action on the other hand induces interesting dynamic characteristics in the system including transition from simple to complex oscillatory behavior via period doubling bifurcation, intermittent chaos, and period adding bifurcation.

A portion of the Poincare bifurcation diagram (intersection of the trajectories with a fixed hyperplane) is shown in Figure 3a. The stable limit cycle that emerges from the Hopf point at the dimensionless proportional gain $K_c = 3.946466$ loses its stability through period doubling. The system then alternates between chaotic regimes through period adding bifur-

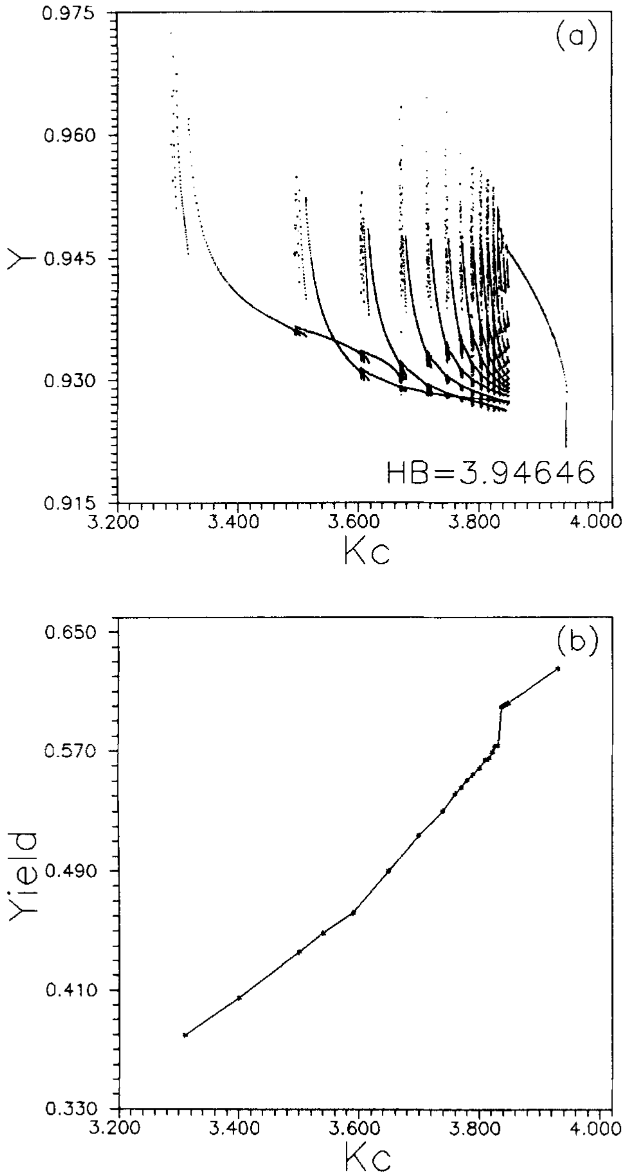

Figure 3. (a) Poincare map (intersection of trajectories with hyperplane $X_B = 0.6313$) of the PI controlled unit (the integral mode gain is $K_i = 0.05$), (b) variations of the yield with the proportional gain.

cation. A larger sequence of periodic regimes of period 1, 2, ... covers the entire chaotic region.

Figure 3b shows the variations of the yield with the proportional controller gain K_c. Starting from the period-1 attractor emerging from the Hopf point it can be seen that as the proportional gain decreases, and the system moves to instability, the yield also decreases.

The presence of chaotic regions in the system does not improve the yield of the reaction. Nevertheless, we will investigate the effects on the yield of periodically forcing a chaotic attractor. The variable chosen for the forcing is the feed temperature, while the center of forcing is a chaotic attractor that corresponds to $K_c = 3.8323$ and lies in the chaotic strip of the autonomous system depicted in Figure 4a. Figures 4b–c show some of the dynamic characteristics of this chosen center of forcing. Both time trace and phase plane are shown. The average yield of the reaction at the chaotic at-

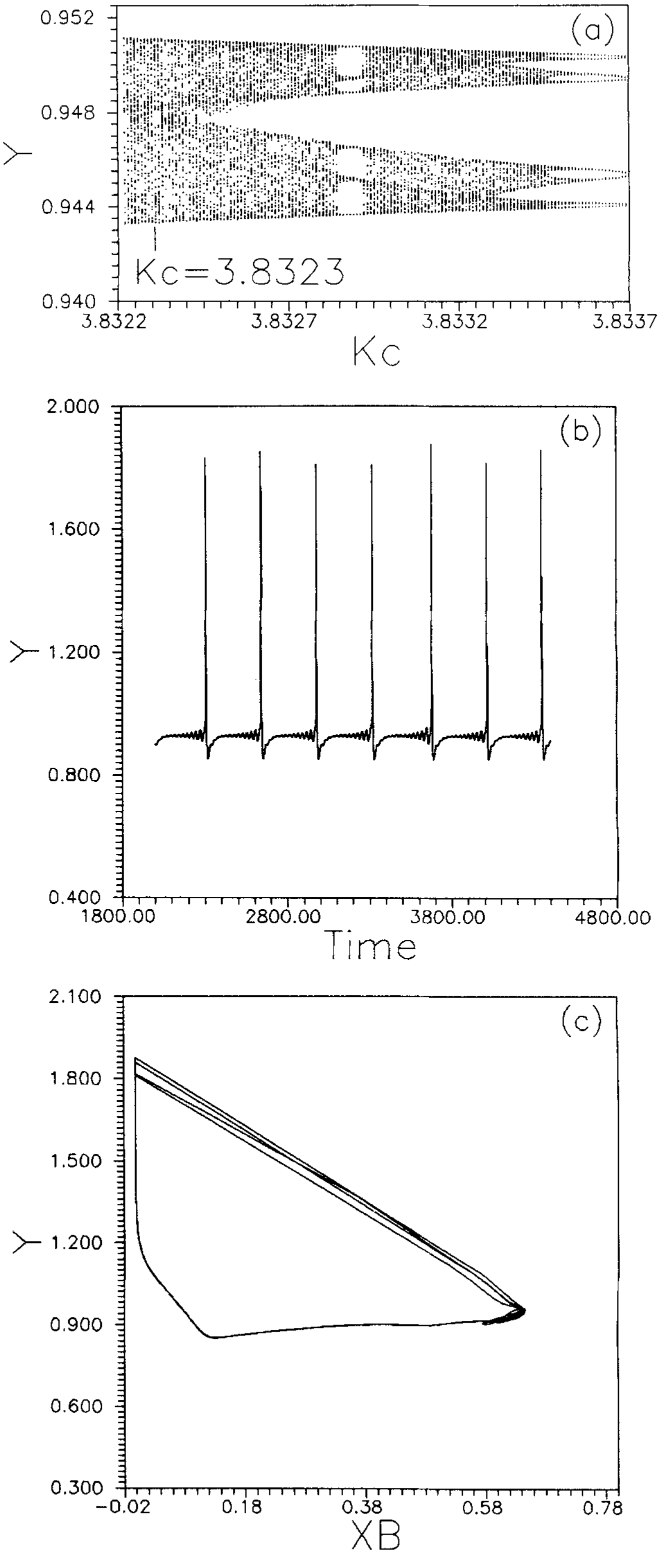

Figure 4. Dynamic characteristics of the center of forcing.

(a) Enlargement of Figure 3 showing the chosen center of forcing; (b) time trace; (c) phase plane.

tractor is 0.57731. It is below the maximum yield 0.62929 that can be obtained by operating the reactor in the stabilized region of the bifurcation diagram (Figure 2).

The investigation of the forced system is suitably carried out using a stroboscopic map. The phase projection of the trajectories are inspected at specific times t (time/heat capacity of the system) multiples of the forcing period, i.e., $t = mT_f$, thereby generating a sequence of points in the phase plane. These points are known as stroboscopic points and the sequence of these points is defined as the iteration of the stroboscopic map. This technique is equivalent to a projection that reduces the dimensionality of the system by one. Instead of following the whole trajectory, stroboscopic points are taken at every forcing period at discrete time intervals. Transient motions appear then on the map as scattered dots, while the emergence of periodic attractor of order n (subharmonic) would be seen as jumps between n fixed points.

It is evident that the simplicity of the presentation is lost when the strobing period, i.e., forcing period, is not exactly determined. High accuracy is then required for the determination of the forcing frequency $w_f = 0.2438777104$ chosen to be the frequency of the period-1 attractor that emerges from the Hopf point (Figure 3a). A shooting method (Kubicek and Marek, 1983) was used for this purpose.

Before presenting the results of the investigation, the mathematical model used in this study is briefly described. The detailed mathematical derivation and the different assumptions were presented by Elnashaie et al. (1977, 1995).

Mathematical Model of the Process

The equations for the dense phase material and energy balances in dimensionless form are given by the following three nonlinear differential equations

$$\frac{1}{Le_A}\frac{dX_A}{dt} = \bar{\beta}(X_{A_f} - X_A) - \alpha_1 \exp\left(-\frac{\gamma_1}{Y}\right)X_A, \quad (1)$$

$$\frac{1}{Le_B}\frac{dX_B}{dt} = \bar{\beta}(X_{B_f} - X_B) + \alpha_1 \exp\left(-\frac{\gamma_1}{Y}\right)X_A - \alpha_2 \exp\left(-\frac{\gamma_2}{Y}\right)X_B, \quad (2)$$

$$\frac{dY}{dt} = \bar{\beta}\left(\bar{Y}_f - Y\right) + \alpha_1\beta_1 \exp\left(-\frac{\gamma_1}{Y}\right)X_A + \alpha_2\beta_2 \exp\left(-\frac{\gamma_2}{Y}\right)X_B \quad (3)$$

X_A, X_B are the dimensionless concentrations of components A and B in the reactor dense phase, and Y is the dimensionless dense-phase temperature. The bubble-phase mass and heat balances equations are assumed at pseudo-steady state because of negligible mass and heat capacities.

The Le_i is the Lewis number of component i. It represents the ratio between the heat capacity of the system and the mass capacity for component i. For gas-solid catalytic systems like the one studied in this article, the mass capacity of a strongly chemisorbed component may exceed the heat capacity, and the Lewis number may vary strongly from unity (Aris, 1975).

The $\alpha_i = (1-\epsilon)\rho_s k_i$ represent the normalized pre-exponential factors for the reactions $A \rightarrow B$ and $B \rightarrow C$, while $\beta_i = -\Delta H_i C_{\text{ref}}/\rho_f C_{pf} T_{\text{ref}}$ for the reaction i is $-\Delta H_i C_{\text{ref}}/\rho_f C_{pf} T_{\text{ref}}$ for the reaction i is the dimensionless overall exothermicity factors, with the ΔH_i (kJ·kg·mol^{-1}) being the overall heats of reaction i.

The $\gamma_i = E_i/RT_{\text{ref}}$ for the reaction i on the other hand is the dimensionless activation energies, and $\bar{\beta}$ represents the reciprocal of the effectiveness residence time of the bed.

Equations 1–3 represent the equations of the autonomous system. The controlled autonomous model is described by Eqs. 1–3 along with the simple control law relating the dimensionless feed temperature $\bar{Y}_f$, chosen as the manipulative variable, to the error $(Y_m - Y)$ through a PI controller with dimensionless gains K_c and K_i

$$\bar{Y}_f = Y_{f_0} + K_c(Y_m - Y) + K_i\int_0^t (Y_m - Y)dt \quad (4)$$

Y_m is the dimensionless temperature (setpoint) corresponding to the desired steady state, and Y_{f_0} is the base value feed temperature.

When the feed temperature is periodically forced, $\bar{Y}_f$ is given by

$$\bar{Y}_f = Y_{f_0} + A_m \sin wt + K_c(Y_m - Y) + K_i\int_0^t (Y_m - Y)dt \quad (5)$$

where A_m and w are the amplitude and frequency of the forcing input.

The autonomous system is obviously the limiting case of the forced system as the forcing amplitude A_m goes to 0. The data used in this investigation are given in Table 1.

The two-phase model used in the investigation retains the main characteristics of the bubbling fluidized bed and has been used successfully to simulate type IV industrial fluid-catalytic cracking units (Elshishini and Elnashaie, 1990, 1993).

Results and Discussion

The first part of the investigation focuses on studying the effects of the forcing amplitude while maintaining the frequency constant. In later sections we study the effects of frequency forcing on the system at constant amplitude.

Effects of forcing amplitude on chaotic attractor

A complete one-parameter stroboscopic bifurcation diagram of the forced system is shown in Figure 5a where the Y-axis represents the stroboscopic dimensionless dense-phase temperature. Figure 5b shows the variations of the yield with the forcing amplitude. It can be seen that starting from the value 0.57731 corresponding to the autonomous chaotic system ($A_m = 0.0$), the yield decreases with the forcing amplitude.

On the scale of Figure 5a, the system looks like an alternation of periodic regimes interrupted by chaotic-like strips, via period adding bifurcation. At high forcing amplitudes and beyond the values of $A_m = 0.105$, the system is fully entrained. The behavior of the system is in fact more complex

Table 1. Data Used in the Model

Normalized preexponent factor for the reaction $A \rightarrow B$, α_1	10^8
Normalized preexponent factor the the reaction $B \rightarrow C$, α_2	10^{11}
Dimensionless overall exothermicity factor for the reaction $A \rightarrow B$, β_1	0.4
Dimensionless overall exothermicity factor for the reaction $B \rightarrow C$, β_2	0.6
Dimensionless activation energy for the reaction $A \rightarrow B$, γ_1	18.0
Dimensionless activation energy for the reaction $B \rightarrow C$, γ_2	27.0
Lewis number of component A, Le_A	1.0
Lewis number of component B, Le_B	0.454545
Feed concentration of component A, X_{A_f}	1.0
Feed concentration of component B, X_{B_f}	0.0
Dimensionless feed temperature to the reactor (basevalue), Y_{f_0}	0.55342072
Set point for the controller, Y_m	0.92955130
Reciprocal of the effective residence time of the bed, $\bar{\beta}$	0.12543050
Dimensionless proportional gain, K_c	3.8323
Dimensionless integral mode gain, K_i	0.05

and an enlargement of this figure is therefore needed to identify the finer structures of the system behavior.

Region R_1: $A_m = 0.00$ to 0.045. Parts of region R_1 of Figure 5a are enlarged in Figure 6a for the amplitude range $A_m \in [0, 0.0020]$. The limiting case of this region is the autonomous chaotic system ($A_m = 0.0$). The system evolves from chaotic to periodic regime by reverse period adding bifurcation (the periodicity of windows decreases as the amplitude increases), with this sequence itself being interrupted by different chaotic strips.

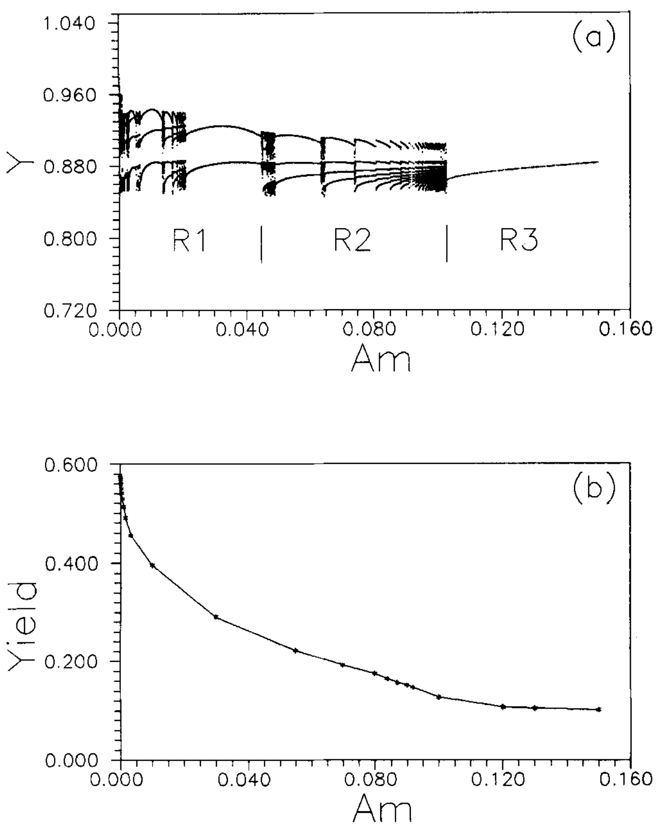

Figure 5. (a) One-parameter stroboscopic Poincare bifurcation diagram; (b) variations of the yield with the forcing amplitude.

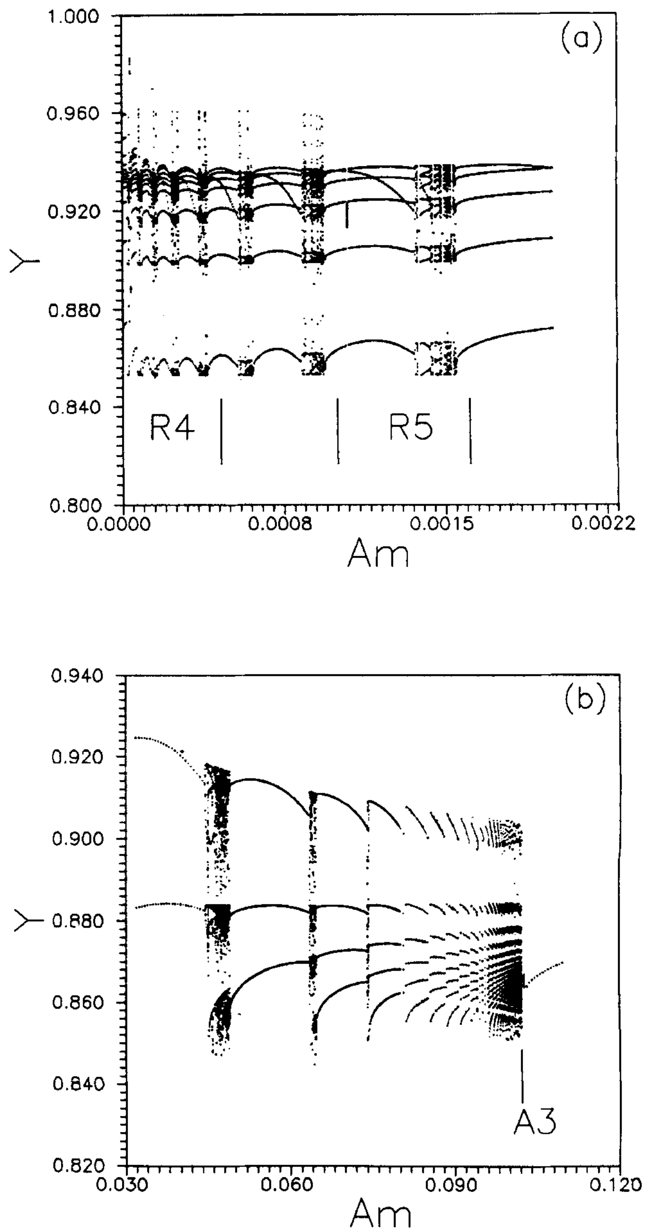

Figure 6. Enlargements of regions of Figure 5: (a) R_1 for the range $A_m \in [0.0, 0.002]$; (b) R_2.

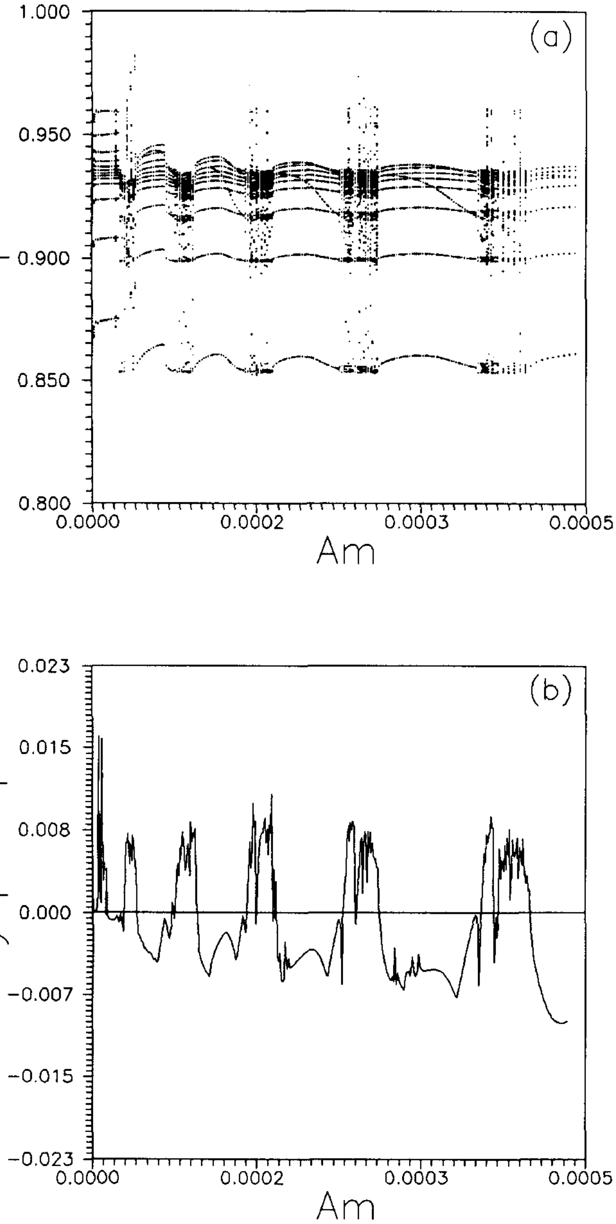

Figure 7. (a) Region R_4 of Figure 6a enlarged; (b) corresponding Lyapunov exponents spectrum.

Regular regimes appear at very small forcing amplitudes.

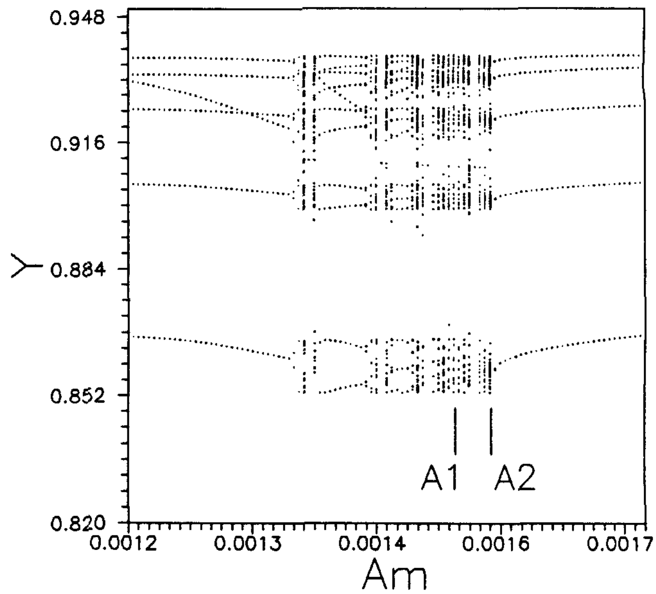

Figure 8. Enlargement of region R_5 of Figure 6a.

Period-6 attractor goes through an alternation of period doubling and intermittent chaos and emerges as a period-5 attractor.

An enlargement of region R_4 of Figure 6a is shown in Figure 7a. It covers the amplitude range $A_m \in [0.00, 0.0005]$. Periodic windows interrupted by strips of chaos start at extremely low values of the forcing amplitude. The frequency locking occurs, therefore, at very small forcing amplitudes. In order to characterize the nature of the emerging attractors Lyapunov exponents are computed for this region. These exponents are the most practical indicators of chaotic behavior of an attractor. A chaotic attractor is characterized by at least one positive Lyapunov exponent. The technique and the algorithm of Wolf et al. (1985) were used to compute these exponents.

Figure 7b shows the maximum component of the Lyapunov exponents as function of the amplitude. The spectrum becomes negative for small values of amplitude ($A_m = 0.00001$) indicating the termination of the chaotic regime and the appearance of a periodic attractor.

The mechanism of transition from chaotic to periodic regime can be shown by examining for instance the region R_5 shown in Figure 6a and enlarged in Figure 8. The system starts its transformation from chaotic to periodic regime with a region of Period-6 attractor starting at $A_m = 0.00134$. As the forcing amplitude A_m increases, the periodic attractor starts to undergo period doubling sequence at $A_m = 0.00135$ leading again to chaos. The new chaotic region again turns periodic at $A_m = 0.00139$ giving a region of P_{10} attractor which bifurcates again to chaos through period doubling and so on. Throughout this region, the system goes through an alternation of periodic windows and chaos and emerges as a Period-5 attractor starting at $A_m = 0.00157$.

The structure of the chaotic attractor is shown by considering the point A_1 of Figure 8 ($A_m = 0.00152$). Dynamic simulations of the chaotic trajectory at this point are shown in Figure 9. The corresponding Lyapunov exponents are 0.003053, −0.034483, −1.023436, −0.623732. The largest component is positive giving proof of the chaotic nature of the attractor.

Periodic Windows and Intermittency. The mechanism of bifurcation from the chaotic regime to the period-5 attractor is investigated by considering the forcing amplitude at the point A_2 of Figure 8 ($A_m = 0.00155$).

Dynamic simulation at this point is shown in Figures 10a–10b. The fifth iterate for the dimensionless dense-phase temperature of $Y(n+5)$ vs. $Y(n)$ (Figure 10b) shows that the curve approaches the diagonal and almost becomes tangent at five distinct points. The chaotic behavior is then destroyed through the mechanism of intermittency as studied by Pomeau and Manneville (1980). The term intermittency refers to oscillations that are periodic for certain intervals (laminar

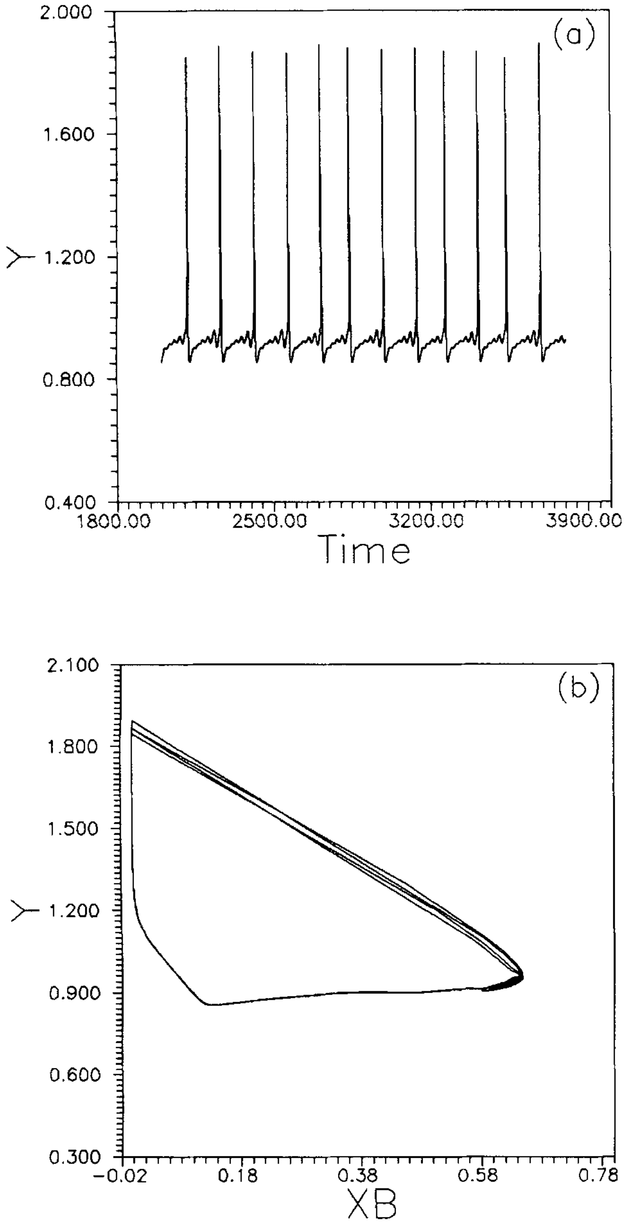

Figure 9. Dynamic characteristics of chaos at the point A_1 ($K_c = 0.0152$) of Figure 8.

(a) Time trace; (b) phase plane.

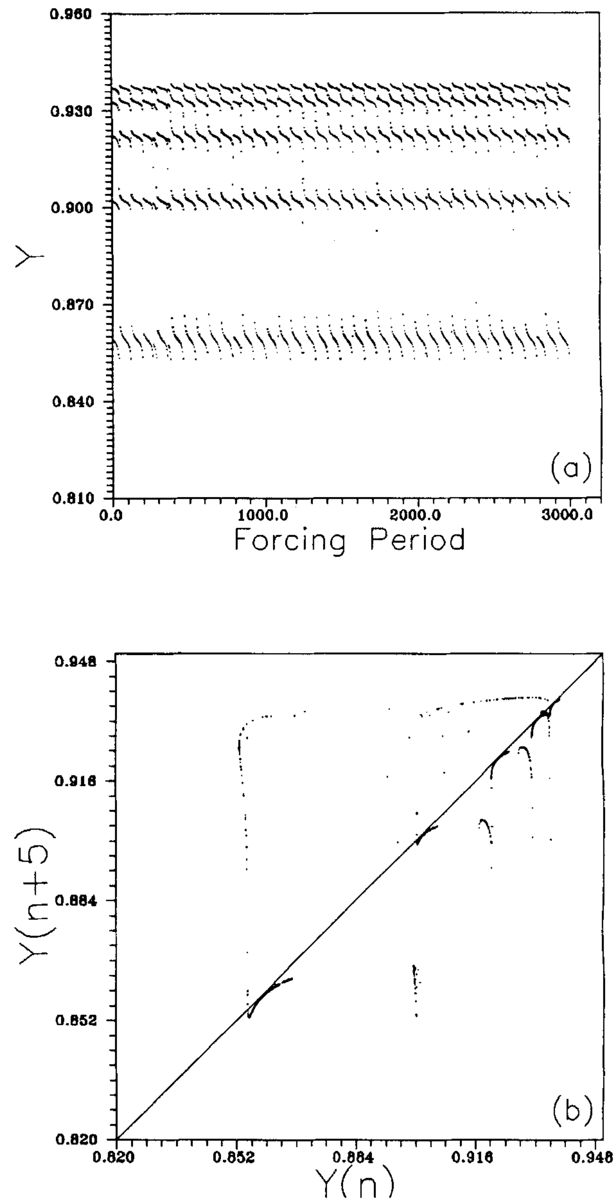

Figure 10. Characteristics of the intermittent chaos at point A_2 ($A_m = 0.0155$) of Figure 8.

(a) Stroboscopic points histogram; (b) fifth-iterate map. The trajectory is tangent to the diagonal in five points.

phase) interrupted by intermittent erratic bursts of periodic oscillations of finite duration. As soon as the curve becomes tangent to the diagonal, the chaotic attractor disappears and each of the five tangent points generates one stable point of the node form and another of the saddle form, i.e., saddle-node bifurcation.

Regions R_2 and R_3: Period Adding of Second Kind. Beyond the point corresponding to $A_m = 0.0450$ (end of region R_1 of Figure 5a), the system alternates between chaotic behavior which emerges from the period doubling sequence and periodic windows as shown in region R_2 enlarged in Figure 6b until the system is fully entrained at high amplitudes (region R_3 of Figure 5a).

The dominant mechanism here is period adding with strips of chaos. This behavior is termed period adding of the second kind and was first analyzed by Holden and Fan (1992) in their study of the autonomous three-dimensional Rose-Hindmarch model for neural activity.

The final bifurcation to the harmonic trajectory (fully entrained) is investigated by choosing the point A_3 (Figure 6b) corresponding to $A_m = 0.102$. Dynamic simulations are shown in Figures 11a and 11b. The maximum Lyapunov exponent is zero further confirming the quasi-periodicity nature of the attractor.

The results of the amplitude forcing of the autonomous catalytic reactor has shown that chaos can be controlled with

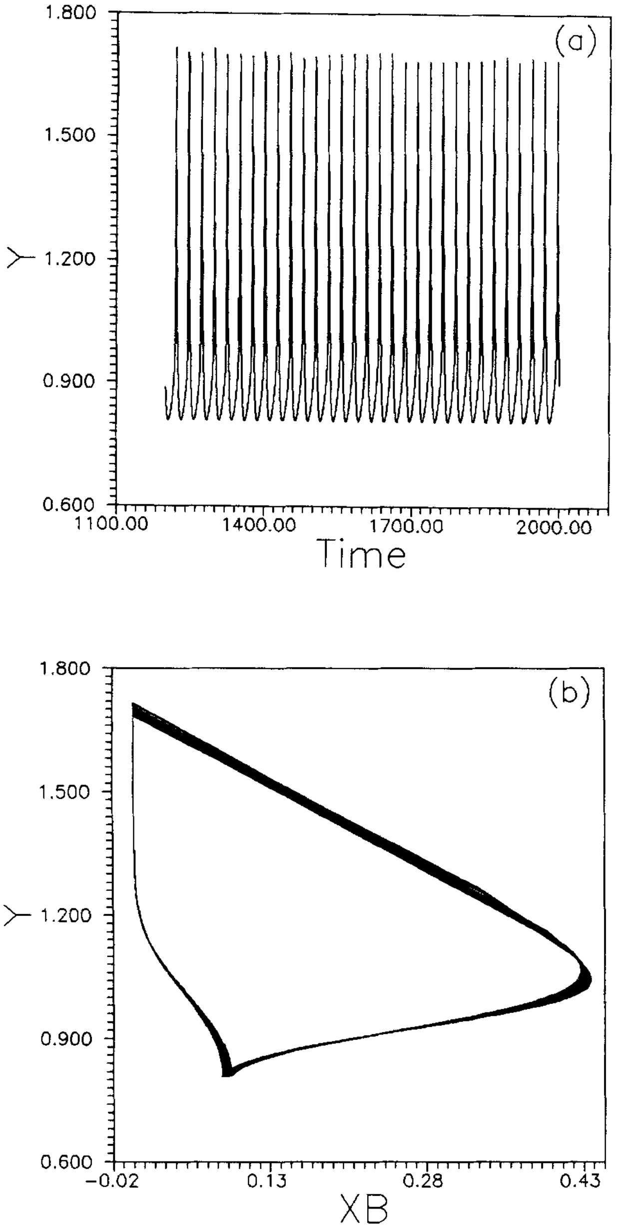

Figure 11. Dynamic characteristics of the point A_3 (A_m = 0.102) of region R_3 (Figure 6b) showing quasi-periodicity.

(a) Time trace; (b) phase plane.

even small values of the forcing amplitude. The practical relevance of these results is that this phenomenon is practically unavoidable given the very small values at which chaotic regimes terminate and periodic regimes appear. The drawback of operating the reactor at the controlled chaotic regions is that the yield deteriorates. However, as it will be shown in the next section, the yield can be improved by an appropriate choice of the forcing frequency.

Effect of frequency forcing on chaotic attractor

This part of the investigation focuses on the effect of the frequency forcing of a chaotic attractor (Point A_1 at K_c = 0.00152) shown in figure 8.

A one-parameter stroboscopic bifurcation diagram is shown in Figure 12 for the point A_1 corresponding to the chaotic attractor. Figure 12a covers the frequency range $w/w_f \geq 1$ while the complementary range $w/w_f \leq 1$ is shown in Figure 12b.

The scale of Figure 12a shows that starting from the chaotic region ($w/w_f = 1$) the system goes through alternation of what looks like chaotic regimes that persist throughout region S_1. At the end of this region at $w/w_f = 1.050$, the chaotic-like oscillations terminate and a periodic regime appears. The pe-

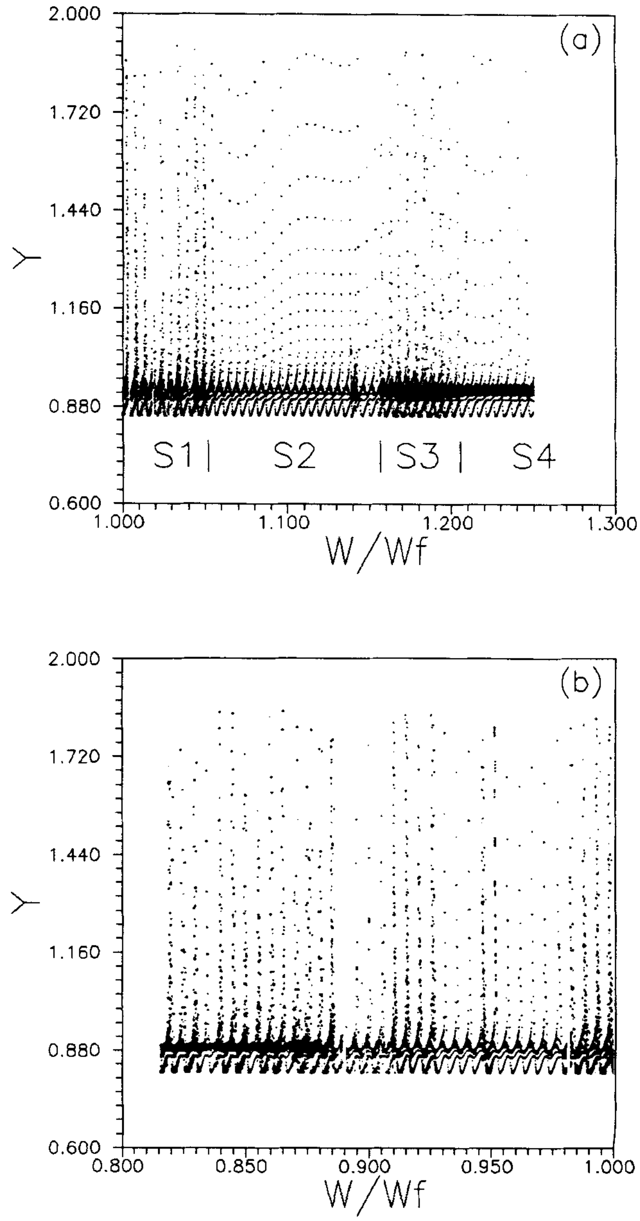

Figure 12. (a) One-parameter stroboscopic Poincare bifurcation diagram at the chaotic attractor (A_m = 0.00152).

(a) Range $w \geq w_f$; (b) Range $w \leq w_f$.

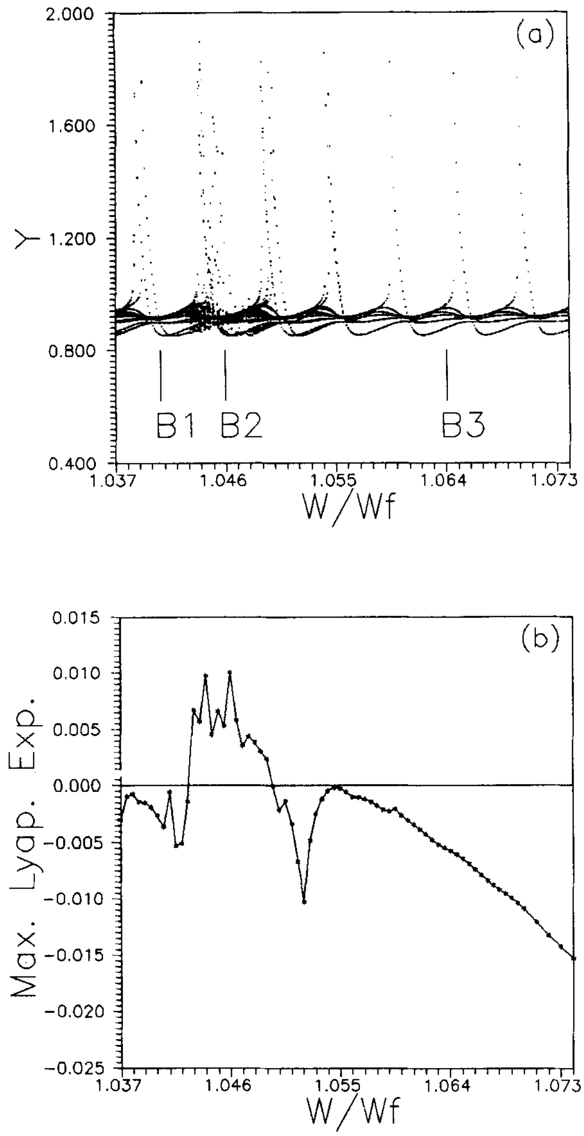

Figure 13. (a) Region of the Poincare diagram for 1.037 ≤ w/w_f ≤ 1.073; (b) corresponding Lyapunov exponents spectrum.

Regular patterns appear as the frequency increases.

riodic regime persists throughout the region S_2. At the end of this region at $w/w_f = 1.155$, a chaotic-like regime appears again and remains throughout the region S_3. At the end of region S_3 at $w/w_f = 1.210$, the chaotic oscillations terminate and periodic regimes start their bifurcation again. The same behavior can be observed for the range $w/w_f \leq 1$, as shown in Figure 12b. The system starts with a chaotic-like regime and as the frequency decreases the system goes through alternation of periodic and chaotic-like regions.

To analyze the finer structure of this forced system, Lyapunov exponents spectrum is computed for a region covering the frequency range $w/w_f \in [1.037, 1.074]$, as shown in Figure 13. This region corresponds to the end of region S_1 and the beginning of region S_2. By analyzing the Lyapunov spectrum, it can be seen that the chaotic-like regimes shown on the scale of Figure 12a (region S_1) are in fact alternations of regions of high periodicity, i.e., point B_1, where the maximum Lyapunov exponent is negative and a chaotic burst, i.e., (point B_2) where the maximum Lyapunov exponent is positive. The system alternates then between these two modes and emerges as a period-6 attractor for $w/w_f = 1.049$.

The dynamic characteristics of the different regions are shown in Figure 14. Both time traces and stroboscopic return points are plotted for the three points B_1, B_2, and B_3. The point B_1 corresponds to a high periodicity attractor, i.e., a P_{10} attractor. Point B_2 corresponds to a chaotic attractor. It can be seen that the points on the corresponding histogram are clearly mixed. Point B_3 is a P_6 attractor, as shown from the histogram of Figure 14a3.

The mechanism of transition from chaotic to periodic behavior is also by intermittency and is similar to the behavior shown in Figure 10b.

Effects on the Yield. In the previous sections the investigations of the amplitude forcing has shown that the yield of the reaction network degrades as the forcing amplitude increases. The variations of the yield with the frequency is, however, not monotonous. Figure 15b shows the results of the investigation of the effect of the frequency on the yield for a small range of the parameter space ($w/w_f \in [1.00, 1.10]$ (Figure 15a). Starting from the initial value of the yield 0.49803 corresponding to the yield of the chaotic strip $A_m = 0.00152$, the yield jumps to a value of 0.61940 for some range of the forcing frequency and then it decreases. The maximum value reached by the yield is even higher than the yield of the center of forcing for the autonomous system (0.57731). It is, however, still lower than the maximum yield 0.62929 that can be obtained by operating the reactor in the stabilized region.

Figure 15c shows the yield corresponding to the unforced system ($A_m = 0.00$, $K_c = 3.8323$), while Figure 15d shows the yield for point $C(w/w_f = 1.031)$ corresponding to the maximum yield obtained with the frequency forcing (Figure 15a). Numerical investigations have thus revealed that the yield of the autonomous chaotic system can be improved by an appropriate selection of the forcing frequency. However, knowing *a priori* the "optimum" frequency is not a trivial task and, to our knowledge, only numerical investigations are suited for this purpose.

Conclusions

Numerical investigation of the effects of periodic perturbations of feed temperature on the dynamics of a two-phase model of a fluidized-bed catalytic reactor has been presented in this article. The investigation has shown that regular regimes can emerge from the original chaotic unperturbed system when the feed temperature is perturbed either in amplitude or in frequency. Different mechanisms for the transition between chaotic regions and periodic windows has been identified and analyzed, including period adding and tangent bifurcation. Numerical investigation has shown that the yield of the desired intermediate product in the consecutive reaction network $A \rightarrow B \rightarrow C$ deteriorates as the forcing amplitude increases, but it was shown that the yield can be improved by an appropriate choice of the forcing frequency. It should be noted, however, that the results of this investigation concern a nominal model of the reactor. The effect of

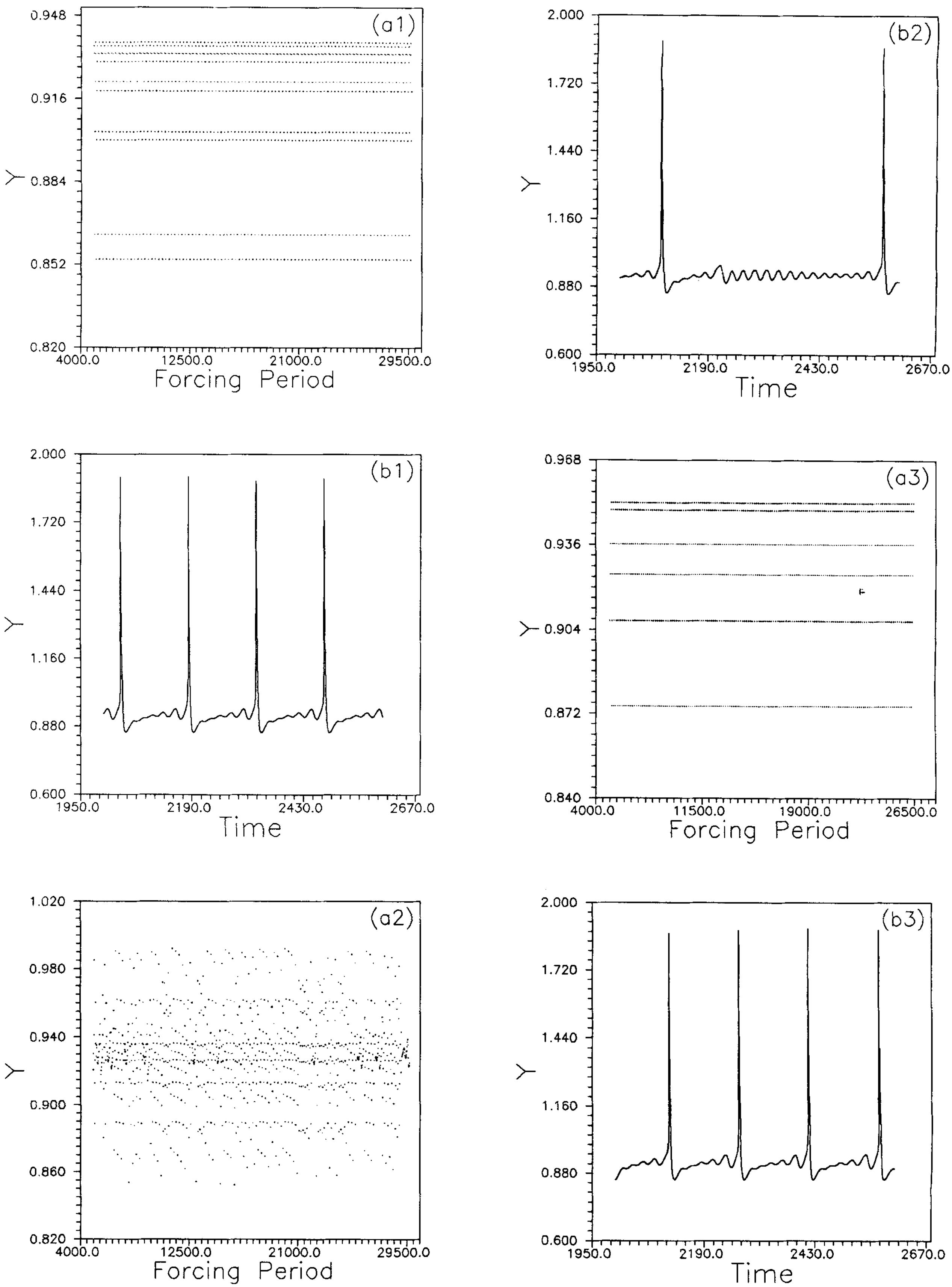

Figure 14. Dynamic characteristics of the region of Figure 13a.

B_1 is an attractor of high periodicity; B_2 is a chaotic attractor; B_3 is a period-6 attractor. (a) Stroboscopic point histogram; (b) time trace.

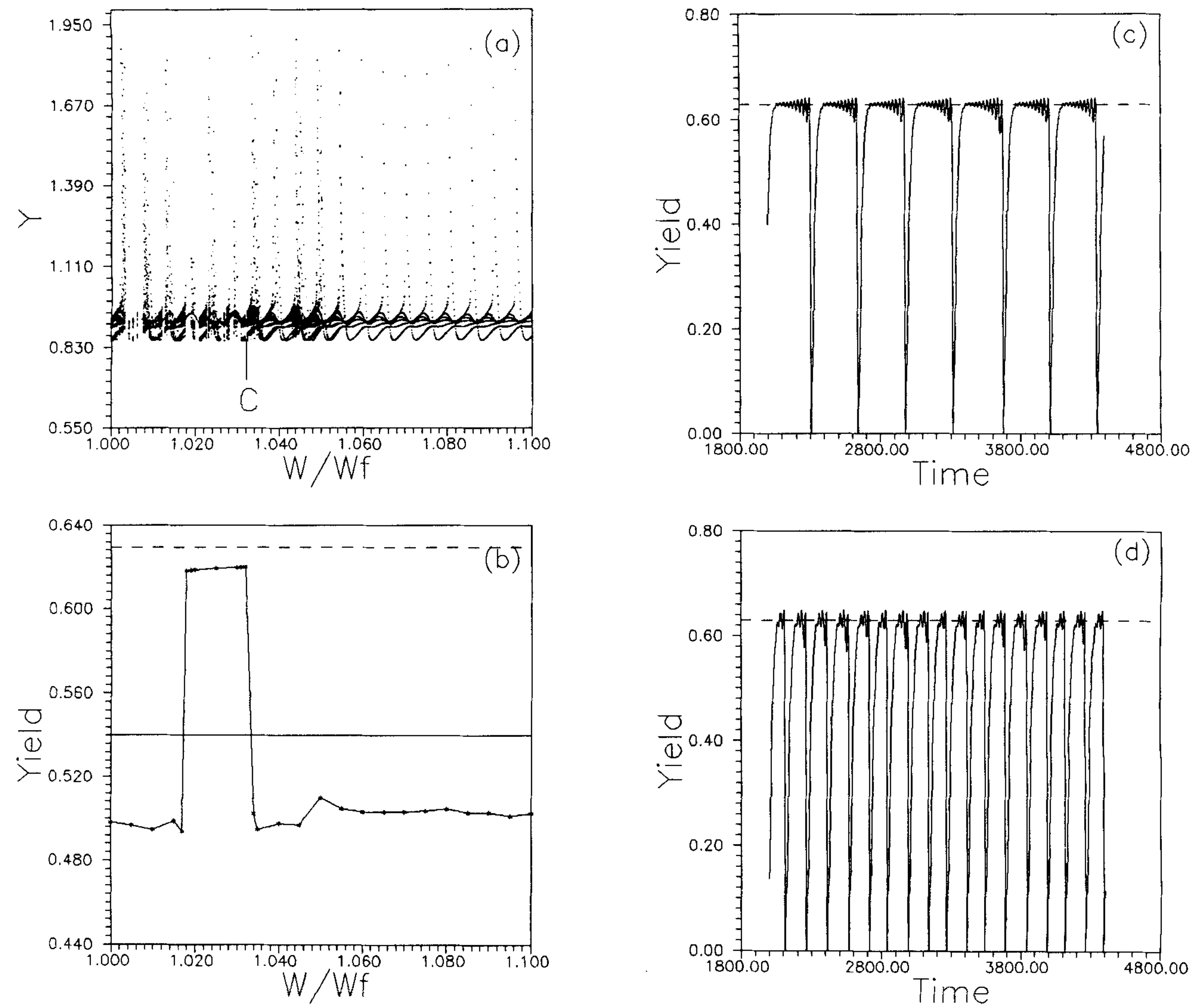

Figure 15. Variations of the yield with the forcing frequency.

(a) Region of Figure 12a enlarged for the range $1.0 \leq w/w_f \leq 1.10$; (b) corresponding yield spectrum; (c) yield at the center of forcing ($A_m = 0.0$, $K_c = 3.8323$); (d) yield at point C of Figure 15a. Dashed line is the yield corresponding to the stable region of Figure 2.

process noise and model uncertainties on the results of these parametric perturbations is an important issue that needs to be investigated.

Notation

A_c = cross-sectional area of the bed occupied by bubble phase, cm^2
A_i = cross-sectional area of the bed occupied by dense phase, cm^2
C_{pf} = specific heat of gas, $kJ \cdot kg^{-1} \cdot K$
C_{ref} = reference concentration, $kmol/cm^3$
E_i = activation energy for reaction i, kJ/kg
G_c = volumetric gas-flow rate in bubble phase, $cm^3 \cdot s^{-1}$
G_i = volumetric gas-flow rate in dense phase, $cm^3 \cdot s^{-1}$
H = expanded bed height for the fluidized bed, cm
k_i = pre-exponential factor for the reaction i, cm^3/kg catalyst s
P_n = attractor of period n
Q_E = mass exchanged coefficient, s^{-1}
T_f = feed temperature, K
T_{ref} = reference temperature, K
ρ_f = density of the gas, $kg \cdot cm^{-3}$
ρ_s = density of the solid catalyst, kg catalyst cm^{-3}

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